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The interpretation of the effect of climbing up electrodes of the dielectric liquids in stationary fields (the Sumoto effect)

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Abstract. An attempt is made to assess quantitatively the dielectrophoretic force in liquids under the action of a nonuniform electric field. Applying 3 kV DC to two partly immersed electrodes of 0.13 mm radius, 1.3 cm apart, acetone would be pulled up to a height of 7.2 cm at the electrodes, if this phenomenon could be ascribed to dielectrophoretic force only.

1. Introduction

The phenomenon of dielectric liquids being pulled up by thin, partly immersed electrodes has been noted by Faraday (1839) and has been rediscovered by Sumoto (1956). The literature on this subject (Schultze 1906, Palmer 1911, Middendorf and Brown 1958, Pohl 1958, Pickard 1961, 1962, Creju *et al* 1968, 1969) contains many experimental data and qualitative interpretations of the phenomenon. In the present work the electrophoretic force and the dielectrophoretic force are considered separately, but no quantitative evaluation of the contribution of electrophoretic forces has been made, because too little is known of the origin and nature of the charged particles.

2. Description of the effect. The main experimental results

In figure 1 the general arrangement is represented by which the effect of the liquid climbing up a single electrode can be obtained, while in figure 2 the general arrangement for the liquid climbing up both electrodes is represented.

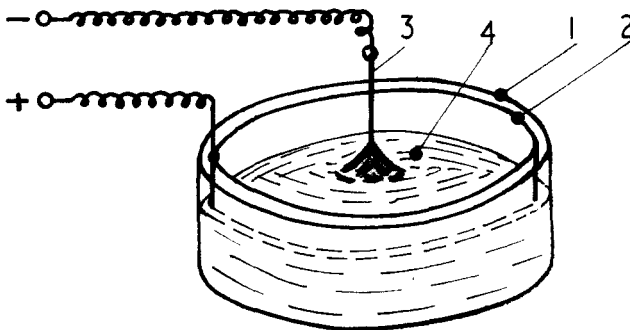


Figure 1. The effect of climbing up a single electrode. 1 glass vessel, 2 cylindrical electrode, 3 coaxial filiform electrode, 4 dielectric liquid.

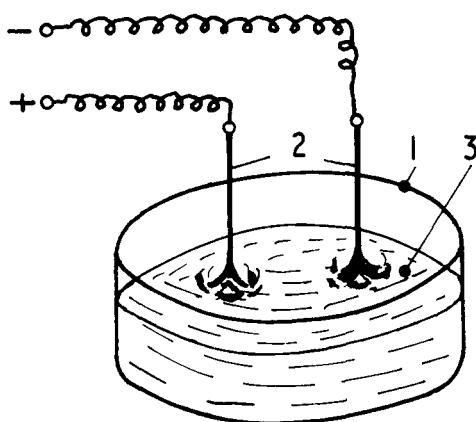


Figure 2. The effect of climbing up both the electrodes. 1 glass vessel, 2 filiform electrodes, 3 dielectric liquid.

The first one has not been met with in the literature on this subject. It has been used by the present author because of the simple symmetry presented by the field.

If the electrodes used are sufficiently thin (with a diameter of the order of tenths of millimetres) and the voltage applied is sufficiently high (of the order of kilovolts), the liquid will climb along the electrodes in the form of cones or 'molehills' (Pickard 1961) attaining a height of the order of millimetres or even centimetres.

An important experimental finding is that, up to a certain level of voltage, the effect does not depend on the polarity of the central electrode of figure 1, being symmetrical for the two electrodes of figure 2. If the voltage is further increased, the 'molehills' or rises of the liquid along the electrodes become asymmetrical, depending on polarity.

For the formulation of a quantitative theory of this effect the first range of voltages is of special interest. We shall therefore mention the main results concerning this range.

Beside the symmetry of the effect, it has been found that the effect increases if:

- (i) the voltage applied to the electrodes increases;
- (ii) the distance between the electrodes decreases;
- (iii) the diameter of the electrodes diminishes;
- (iv) the permittivity of the insulating liquid is greater;
- (v) the temperature of the liquid is higher.

The second range of voltages presents other interesting aspects as well, among which we mention:

- (vi) with the increase of voltage the symmetry of the effect from the first range is impaired (figure 3(a)) by the diminution and final disappearance of one molehill (as a rule at the anode) and the simultaneous increase of the other (at the cathode)(figure 3(b)). This change takes place simultaneously with the variation of voltage.
- (vii) The above phase also changes in time, so that in the course of several hours—at a distance of the order of centimetres between the electrodes—the molehill or rise of cathodic liquid also diminishes in height down to zero (the voltage being kept constant).
- (viii) the effect is accompanied by a transfer of metal between the two electrodes and is strongly influenced by the purity of the liquid.

The literature on the subject presents numerous other observations, which, however, are considered as being but of secondary importance for the purpose of the present paper. The theory developed here quantitatively expresses the results (i) to (v) and explains the symmetry of the effect in the case of a change of polarity.



Figure 3. The profile of the dielectric liquid. (a) The first range of voltages; (b) the second range of voltages.

3. The theory of the effect of the liquid climbing up a single electrode

We shall assume the electrodes to be cylindrical, coaxial, straight and sufficiently long for the field to be considered practically plane-parallel in their median zone, in which the free surface of the liquid is also situated.

In the first range of voltages the reference liquid remains nonconductive (or practically so), so that the forces exerted by the electric field are due exclusively to the local inhomogeneity of the medium and to the local nonuniformity of the field (Abraham and Becker 1932):

$$\vec{f}_v = -\frac{1}{2}E^2 \text{grad } \epsilon + \frac{1}{2} \text{grad} \left(E^2 \frac{\partial \epsilon}{\partial \tau} \right) = \frac{1}{2} \tau \text{grad} \left(\frac{\partial \epsilon}{\partial \tau} E^2 \right) \tag{1}$$

in which the magnitudes (of known significance) are defined in a reference system which is at rest as compared to the medium consideration. In the literature on this subject these are sometimes called ‘dielectrophoretic’ forces (Pohl 1958).

The \vec{f}_v forces, localized in the free surface of the fluid—which is assimilated to an inhomogeneous layer of rapid but continuous variation of magnitudes and whose amplitude tends towards zero—lead to the specific surface force (Daba 1970)

$$\vec{f}_s = \frac{1}{2} D_n^2 \text{grad}_s \frac{1}{\epsilon} - \frac{1}{2} E_t^2 \text{grad}_s \epsilon + \frac{1}{2} \text{grad}_s \left(E^2 \frac{\partial \epsilon}{\partial \tau} \right).$$

Since the surface of the liquid is a field surface, it has the expression

$$\vec{f}_s = \frac{1}{2} \left(\epsilon - \epsilon_0 - \frac{\partial \epsilon}{\partial \tau} \tau \right) E^2 \vec{n}_{12} \tag{2}$$

where \vec{n}_{12} is the unit vector normal to the surface and directed toward the second space (space 2, ie vacuum), and $E = E_t$ is the local value of the field intensity.

In the range of validity of the Clausius–Mosotti equation, where

$$\frac{\partial \epsilon}{\partial \tau} \tau = \frac{(\epsilon - \epsilon_0)(\epsilon + 2\epsilon_0)}{3\epsilon_0} > \epsilon - \epsilon_0$$

the force \vec{f}_s is directed towards the fluid, hence it exerts a compression on it. This result

is in disagreement with the interpretation given in the literature on the subject, according to which the surface forces 'draw' up the liquid, raising it along the electrodes.

But the forces of the field act also on the volume of the liquid; they are localized in the zones of nonuniform field (see equation (1)), being directed towards the central electrode where the field is more intense. Here they lead to an 'electrical' excess pressure which is added to the local hydrostatic pressure. Its value at a certain point in the field is obtained by the integration of the effects of these forces from a point within the fluid, but situated outside the field, up to the point under consideration (Abraham and Becker 1932)†

$$\Delta p_e = \frac{1}{2} \frac{\partial \epsilon}{\partial \tau} \tau E^2. \quad (3)$$

As may be seen, it does depend solely on the local value of the field and on the medium. For its actual calculation, the electric field distribution must be known.

This volumetric excess pressure is mainly responsible for the electrohydrodynamic effect studied.

The hypothesis of the plane-parallel field between the electrodes implies the infinite length of the latter. It permits the approximation of the real field to a known one, of simple cylindrical symmetry (figure 4).

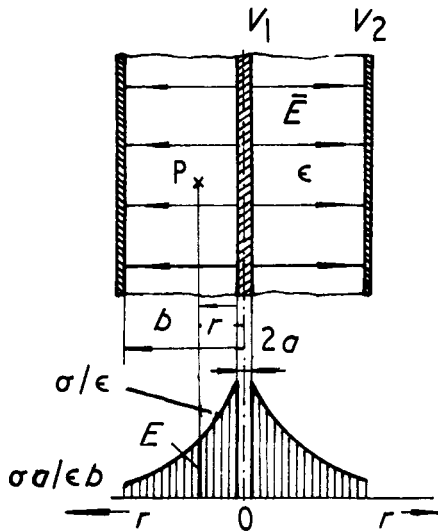


Figure 4. The field between two coaxial, cylindrical, straight electrodes.

At the point P in the liquid, situated at the distance r from the axis, the field has the known value

$$E = \frac{\sigma a}{\epsilon r} \quad (4)$$

where σ is the density of the surface charge of the central electrode and a is its radius.

The field, therefore, varies hyperbolically in the space between the electrodes, from a maximum value at points on the surface of the central electrode down to the minimum value at points on the interior surface of the external electrode.

† The excess pressure calculated in this way has been checked experimentally (Hakim and Higham 1962).

The system of the two electrodes, supplied with the voltage U , is equivalent to a cylindrical condenser. As the capacitance of its unit of length is written as

$$C^* = \frac{2\pi\epsilon}{\ln(b/a)} = \frac{2\pi a\sigma}{U}$$

we have

$$\sigma = \frac{\epsilon U}{a \ln(b/a)}$$

and, therefore, E from (4) obtains the following form :

$$E = \frac{U}{r \ln(b/a)}. \tag{5}$$

As may be seen, the local value of the field increases if either the voltage is increased (the other magnitudes remaining constant), or if the distance between the electrodes is diminished, or electrodes of smaller diameters are chosen. The local value of the field has its highest value at the surface of the internal electrode.

These data given, the local excess pressure (3) at point P becomes

$$\Delta p_e = \frac{1}{2} \frac{\partial \epsilon}{\partial \tau} \tau \frac{U^2}{\{\ln(b/a)\}^2 r^2}. \tag{6}$$

This excess pressure—of the same value at all points on the cylindrical surface which are coaxial with the system and which have the radii r —produces a rise of the liquid and, therefore, a local disturbance of its free surface. The height h of this rise results from the equilibrium condition

$$\Delta p_e + f_s - t = \tau g h \tag{7}$$

where τ is the mass density of the liquid used, g the acceleration due to gravity and t the local surface tension.

Or, with (2), (5) and (6)

$$h = \frac{1}{\tau g} \left(\frac{\epsilon - \epsilon_0}{2} \frac{U^2}{\{\ln(b/a)\}^2 r^2} - t \right) \tag{8}$$

an expression which shows that the rise of the liquid around the central electrode decreases much more rapidly than the value of the field (figure 5).

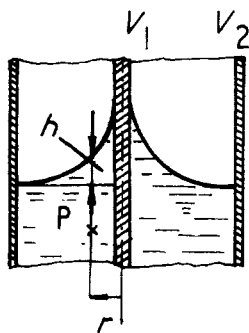


Figure 5. The profile of the free surface of dielectric liquid in electric field.

The expression (8) satisfies the results (i) to (v), as well as the independence of polarity. It is true if the condition used as hypothesis in the calculations is assured, that is, the penetration depth of the electrodes in the liquid should be several times greater at least than the distance between them. When this condition is satisfied, the depth of penetration will not appreciably influence the effect.

Consequently, in order to obtain pronounced effects, light liquids must be used, whose permittivities should be high and whose surface tensions should be as low as possible.

4. The theory of the effect of the liquid climbing up both electrodes

Starting from the same hypothesis as in § 3, the result is obtained that in the free surface of the liquid (the field surface) the same surface force (2) is active.

The electric field distribution will then be identical with that of a system of two infinitely long, straight, cylindrical conductors supplied with a constant voltage $U = V_1 - V_2$. It is known that the equipotential lines of this system are circles of Apollonius, the field lines being orthogonal to these (figure 6).

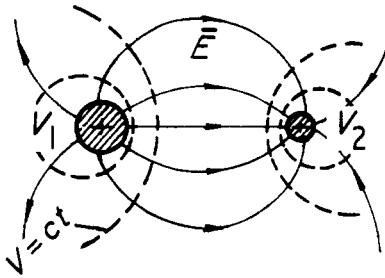


Figure 6. The field spectrum at a bifilar system.

As the distribution of the free electric charge on the surface of the two electrodes is not known, the method of electric images has been used for the calculation of the field. Two linear charge distributions of densities γ and $(-\gamma)$ have been used as images. But because, in our case, the radii of the electrodes are much smaller than the distance between their axes, the images may be considered, to a good approximation, as being situated in the two axes (figure 7).

With this assumption the potentials created by the images at distances a_1 and a_2 from γ and $(-\gamma)$ (the potentials of the electrodes) respectively, are

$$V_1 = \frac{\gamma}{2\pi\epsilon} \ln \frac{d}{a_1}$$

$$V_2 = \frac{\gamma}{2\pi\epsilon} \ln \frac{a_2}{d}$$

and hence

$$U = V_1 - V_2 = \frac{\gamma}{2\pi\epsilon} \ln \frac{d^2}{a_1 a_2}. \quad (9)$$

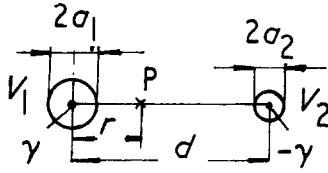


Figure 7. Concerning the field calculation.

At a certain point between the electrodes (point P), situated in the plane of the images, the potential is

$$V = \frac{\gamma}{2\pi\epsilon} \ln \frac{d-r}{r}$$

and the field intensity

$$E = -\frac{\partial V}{\partial r} = \frac{\gamma}{2\pi\epsilon} \frac{d}{r(d-r)}$$

Or, replacing γ from (9)

$$E = \frac{U}{\ln(d^2/a_1a_2)} \frac{1}{r\{1-(r/d)\}} \tag{10}$$

a relation which shows that the local value of the field increases together with the increase of the voltage, with the diminution of the distance between the electrodes or with the diminution of their radii, reaching its highest value on the surface of the electrode with the smaller radius a_2 . Its minimum value is localized at a point whose position is obtained from the condition

$$\frac{\partial E}{\partial r} = 0$$

which leads to

$$r = \frac{d}{2}$$

In the experiments described in the literature on the subject the two electrodes used had the same diameter, that is, $a_1 = a_2 = a$, in which case the field E presents a symmetrical variation with respect to the two electrodes. This variation is qualitatively presented in figure 8.

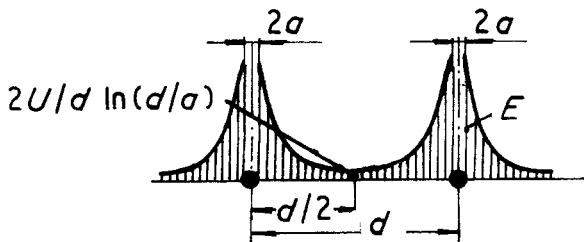


Figure 8. The approximate variation of the field value E .

Consequently, at P the excess pressure (3) will have the value

$$\Delta p_e = \frac{1}{2} \frac{\partial \epsilon}{\partial \tau} \tau \frac{U^2}{4 \{\ln(d/a)\}^2} \frac{1}{r^2 \{1 - (r/d)\}^2}. \quad (11)$$

The altitude h of the local rise of the fluid will result from the same condition (7) as under (§ 3), that is

$$h = \frac{1}{\tau g} \left(\frac{\epsilon - \epsilon_0}{8} \frac{U^2}{\{\ln(d/a)\}^2} \frac{1}{r^2 \{1 - (r/d)\}^2} - t \right). \quad (12)$$

Observations on the climbing of a liquid on a single electrode agree with the values calculated from equation (12).

5. Conclusions

On the basis of the above one may conclude that the main factor responsible for the effect of the rise of liquids along the electrodes is formed by the forces due to the non-uniformity of the field and to the inhomogeneity of the medium. It has often been assumed that the term $\frac{1}{2} \text{grad}\{E^2(\partial\epsilon/\partial\tau)\tau\}$ which is the electrostrictive component, can be neglected. This simplification implies, however, that only surface forces act on the liquid in an electric field, and this is considered to be a wrong interpretation of the mechanism of the effect.

The second factor is constituted by the forces exerted by the field on the charged microparticles that appear in the liquid in the second range of voltages. These forces, which are also known as 'electrophoretic' forces (Pohl 1958), depend on polarity, influence the effect of the dielectrophoretic forces mentioned above, and determine the subsequent evolution of the phenomenon (see results (vi)–(viii)).

Appendix

In the following, the field intensity and the maximum height of the rise h in the case of the arrangement with two electrodes from figure 2 are calculated.

Supposing that

$$U = V_1 - V_2 = 3 \text{ kV}$$

$$a_1 = a_2 = a = 0.13 \text{ mm}$$

$$d = 1.3 \text{ cm}$$

also supposing that acetone is chosen as dielectric—for which the literature provides experimental results (Pickard 1961)—that is

$$\tau g = 0.79 \text{ kg dm}^{-3}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = 21.2$$

and that in the first approximation the pressure t due to the surface tension will be

neglected, then for $r = a$, hence $r/d = 10^{-2} \ll 1$, from (10) we have the results:

$$E_{\max} = E_{r=a} = \frac{U}{2a \ln(d/a)} = \frac{3}{2 \times 0.13 \times 10^{-1} \ln 10^2} = 25 \text{ kV cm}^{-1}$$

and

$$E_{\min} = E_{r=d/2} = \frac{2U}{d \ln(d/a)} = \frac{2 \times 3}{1.3 \ln 10^2} = 1 \text{ kV cm}^{-1} \ll E_{\max}.$$

From (12):

$$\begin{aligned} h_{\max} = h_{r=a} &= \frac{\epsilon - \epsilon_0}{8\tau g} \left(\frac{U}{a \ln(d/a)} \right)^2 = \frac{1}{2\tau g} (\epsilon - \epsilon_0) E_{\max}^2 \\ &= \frac{1}{2 \times 0.79 \times 9.81 \times 10^3} \frac{21.2 - 1}{4\pi \times 9 \times 10^9} (25 \times 10^5)^2 = 7.2 \times 10^{-2} \text{ m} \\ &= 7.2 \text{ cm.} \end{aligned}$$

Owing to the presence of the ions in the liquid and to the simplifying hypotheses adopted in the calculations, the value obtained is greater than the one experimentally observed.

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